

Completing the Square

Given an expression in the form $ax^2 + bx + c$ where $a \neq 0$, this is the step-by-step process for completing the square to write the expression in **vertex form**. $f(x) = \pm a(x-h)^2 + k$

<u>Description</u>	<u>Algebra</u>
1. Group variable terms away from the constant. <i>(to build squares)</i>	$ax^2 + bx + c$ $(ax^2 + bx) + c$
2. Factor 'a' out of the two terms in the parentheses. <i>(Dividing pieces equally into the square)</i>	$a(x^2 + \frac{b}{a}x) + c$
3. Add $(\frac{1}{2} \cdot \frac{b}{a})^2$ inside the parentheses, but this means keeping it the same by subtracting $a \cdot (\frac{b}{2a})^2$ from the constant. <i>(Completing the square by MOVING units from "extra" group to squares)</i>	$a(x^2 + \frac{b}{a}x + (\frac{b}{2a})^2) + c - a(\frac{b}{2a})^2$
4. Rewrite the expression in vertex form. <i>(How many perfect squares with equal dimensions & ADJUSTMENT to area)</i>	$a(x + \frac{b}{2a})^2 + (c - a(\frac{b}{2a})^2)$

Example: Complete the square to write in **vertex form**.

$$\begin{aligned}
 A(x) &= (3x^2 + 12x) + 17 \\
 &= 3(x^2 + 4x + 2^2) + 17 - 3(4) \\
 &= 3(x+2)^2 + 17 - 12 \\
 &= 3(x+2)^2 + 5
 \end{aligned}$$

Three squares w/dimensions of $(x+2)$ with 5 units EXTRA.

