Inverse of a Function

An inverse of a function is the relation formed when the independent variable is exchanged with the dependent variable. If the inverse of a function is itself a function, it is then called an Inverse Function ("undo" function).
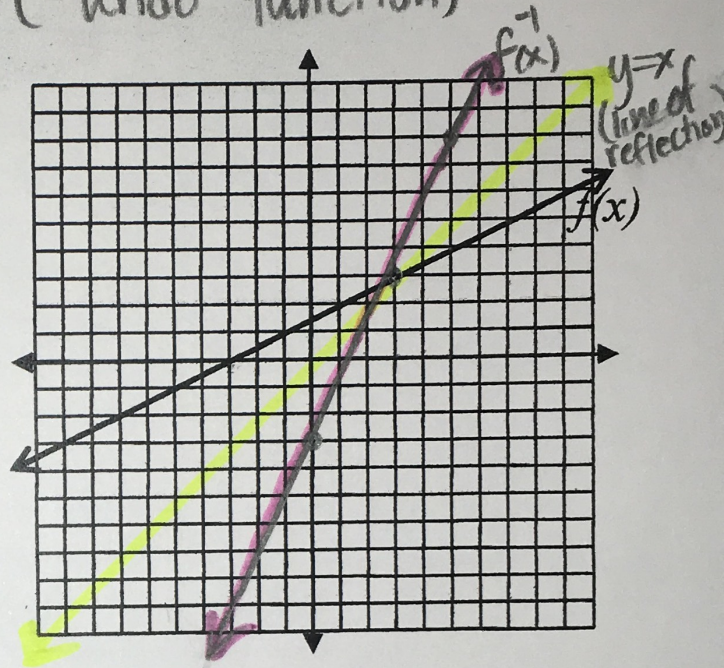
(*Some f(x) may need a restricted domain for inverse to be a function)

Examples:

Given the graph of the function, graph the inverse.

- Reflection over line y=x (same shape)

- $(x,y) \rightarrow (y,x)$

$(-3,0) \rightarrow (0,-3)$ [f(x) → f⁻¹(x)]

$(7,5) \rightarrow (5,7)$

If this were represented in a table:

x	f(x)
−1	1
1	2
−3	0
−7	−2
5	4

→

x	f⁻¹(x)
1	−1
2	1
0	−3
−2	−7
4	5

$f^{-1}(x)$ input is the output of f(x); output of $f^{-1}(x)$ gives original input (from f(x))

Find the equation of the inverse function.

- Switch x & y
- Solve for y

$$f(x) = \frac{1}{2}(x + 5) - 1$$

inverse → $x = \frac{1}{2}(y+5) - 1$

$+1 \qquad\qquad +1$

$2 \cdot (x+1) = \left(\frac{1}{2}(y+5)\right) \cdot 2$

$2(x+1) = y+5$

$\qquad -5 \qquad -5$

$2(x+1) - 5 = y$

$$\boxed{f^{-1}(x) = 2(x+1) - 5}$$