End Behavior of Rational Functions

The end behavior of a rational function is determined, in part, by the relationship between the degrees of the polynomials in the numerator and the denominator.

Let *n* be the degree of the numerator and *d* the degree of the denominator, if

	Type of Asymptote	How to find the asymptote
$n < d$		
$n = d$		
$n > d$		

Examples:

1. $f(x) = \frac{x^2+10x+16}{(x+3)(x-2)(x+5)}$ Asymptote: as $x \to -\infty$, $f(x) \to$ _____ as $x \to \infty$, $f(x) \to$ _____	2. $f(x) = \frac{x(x+1)}{(x-7)}$ Asymptote: as $x \to -\infty$, $f(x) \to$ _____ as $x \to \infty$, $f(x) \to$ _____
3. $f(x) = \frac{(x+2)(x+1)}{(x-5)(x+71)}$ Asymptote: as $x \to -\infty$, $f(x) \to$ _____ as $x \to \infty$, $f(x) \to$ _____	4. $f(x) = \frac{x(2x-3)}{(3-x)(x+6)}$ Asymptote: as $x \to -\infty$, $f(x) \to$ _____ as $x \to \infty$, $f(x) \to$ _____
5. $f(x) = \frac{-3x^2-2x-8}{(x-4)}$ Asymptote: as $x \to -\infty$, $f(x) \to$ _____ as $x \to \infty$, $f(x) \to$ _____	6. $f(x) = \frac{3x^3}{2x^4+9}$ Asymptote: as $x \to -\infty$, $f(x) \to$ _____ as $x \to \infty$, $f(x) \to$ _____