Composites and Proving Inverses

A composite function is a function that operates on another function.

A compound function is written as nested functions, in the form $f(g(x))$.

This is also sometimes written as $f \circ g(x)$.

Use $f(x) = 2x + 1$, $g(x) = x^2 - 2$, & $h(x) = -3x + 7$ to determine the following.

$f(h(x)) = f(-3x+7)$
$= 2(-3x+7)+1$
$= -6x+14+1$
$f(h(x)) = -6x+15$

$g(f(x)) = g(2x+1)$
$= (2x+1)^2 - 2$
$= 2x(2x+1)+1(2x+1)-2$
$= 4x^2+2x+2x+1-2$
$g(f(x)) = 4x^2+4x-1$

$h \circ g(x) = h(x^2-2)$
$= -3(x^2-2)+7$
$= -3x^2+6+7$
$h(g(x)) = -3x^2+13$

To algebraically determine that a function is the inverse of another function, it is necessary to show that $f(g(x)) = g(f(x)) = x$

inverses "undo" each other (they both simplify to ORIGINAL INPUT!)

Determine if $f(x)$ *and* $g(x)$ are inverses.

Ex 1: $f(x) = (x+1)^2$ & $g(x) = \sqrt{x} - 1$

$f(g(x)) = ((\sqrt{x}-1)+1)^2$
$= (\sqrt{x})^2$
$= x$ ✓

$g(f(x)) = \sqrt{(x+1)^2} - 1$
$= x+1-1$
$= x$ ✓

Inverses because $f(g(x)) = g(f(x)) = x$

(they "undo" each other)

Ex 2: $f(x) = 3x - 2$ & $g(x) = \frac{x+2}{3}$

$f(g(x)) = 3\left(\frac{x+2}{3}\right) - 2$
$= x+2-2$
$= x$ ✓

$g(f(x)) = \frac{(3x-2)+2}{3}$
$= \frac{3x}{3}$
$= x$ ✓

Inverses because $f(g(x)) = g(f(x)) = x$

(they "undo" each other)