End Behavior of Rational Functions

The end behavior of a rational function is determined, in part, by the relationship between the degrees of the polynomials in the numerator and the denominator.

Let n be the degree of the numerator and d the degree of the denominator, if

	Type of Asymptote* (*End behavior Asymptote → only behaves as asymptote @ extreme end values)	How to find the asymptote
PROPER: $n < d$	Horizontal Asymptote: $y=0$	ALL proper rational functions approach ZERO at the ends.
IMPROPER: $n = d$	Horizontal Asymptote: $y=\#$	These rational functions approach the value of the Ratio of lead coefficients
IMPROPER: $n > d$	Non-Horizontal Asymptote → $q(x)$ (SLANT asymptote occurs when $n=d+1$)	To APPROXIMATE $q(x)$, use the lead terms & divide (To be more precise, divide ACTUAL polynomials to find $q(x)$) The end behavior of the polynomial ($q(x)$) determines the end behavior.

Examples:

1. $f(x)=\frac{x^2+10x+16}{(x+3)(x-2)(x+5)}$ Proper HA: $y=0$ Asymptote: as $x\to-\infty$, $f(x)\to$ 0 as $x\to\infty$, $f(x)\to$ 0	2. $f(x)=\frac{x(x+1)}{(x-7)}$ Improper ($n>d$) $q(x)\approx\frac{x^2}{x}\approx x$ Asymptote: as $x\to-\infty$, $f(x)\to$ $-\infty$ as $x\to\infty$, $f(x)\to$ ∞
3. $f(x)=\frac{(x+2)(x+1)}{(x-5)(x+71)}$ Improper ($n=d$) $\frac{x^2}{x^2}=1$ HA: $y=1$ Asymptote: as $x\to-\infty$, $f(x)\to$ 1 as $x\to\infty$, $f(x)\to$ 1	4. $f(x)=\frac{x(2x-3)}{(3-x)(x+6)}$ Improper ($n=d$) $\frac{2x^2}{-x^2}=-2$ HA: $y=-2$ Asymptote: as $x\to-\infty$, $f(x)\to$ -2 as $x\to\infty$, $f(x)\to$ -2
5. $f(x)=\frac{-3x^2-2x-8}{(x-4)}$ Improper ($n>d$) $q(x)\approx\frac{-3x^2}{x}\approx-3x$ Asymptote: as $x\to-\infty$, $f(x)\to$ ∞ as $x\to\infty$, $f(x)\to$ $-\infty$	6. $f(x)=\frac{3x^3}{2x^4+9}$ Proper HA: $y=0$ Asymptote: as $x\to-\infty$, $f(x)\to$ 0 as $x\to\infty$, $f(x)\to$ 0