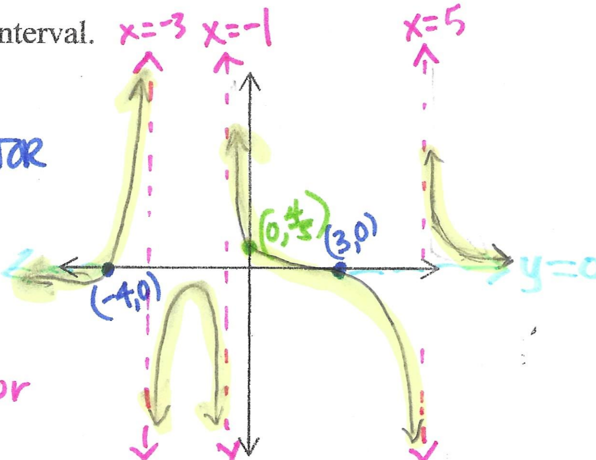
Graphing Rational Functions

To sketch the graph of a rational function, determine the end behavior, find all of the critical points (roots, asymptotes or domain restrictions, & y-intercept), and use a sign line to determine where the function is located on any interval.



1. $f(x) = \frac{(x-3)(x+4)}{(x+1)(x-5)(x+3)}$

x-intercepts(s): ($y=0$ → where is NUMERATOR $=0$?)

$0=(x-3)(x+4)$

$x-3=0$; $x+4=0$

$x=3, -4$ $(3,0)$ & $(-4,0)$

y-intercept: ($x=0$)

$f(0)=\frac{(-3)(4)}{(1)(-5)(3)} = \frac{-12}{-15} = \frac{4}{5}$ $(0, \frac{4}{5})$

Vertical asymptote(s): where denominator $=0$

$0=(x+1)(x-5)(x+3)$

$x+1=0$; $x-5=0$; $x+3=0$

$x=-1$; $x=5$; $x=-3$

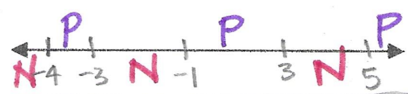
Circle: Proper or Improper

End behavior asymptotes*:

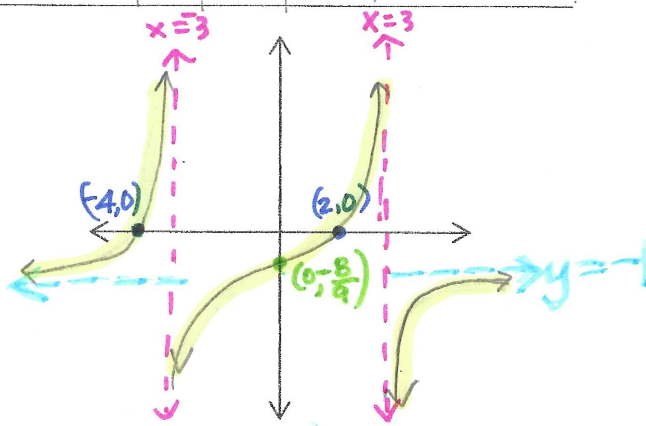
as $x \to -\infty$, $f(x) \to$ 0

as $x \to \infty$, $f(x) \to$ 0

Create a sign line, then complete the graph above.



$x-3$	N	N	N	N	P	P
$x+4$	N	P	P	P	P	P
$x+1$	N	N	N	P	P	P
$x-5$	N	N	N	N	N	P
$x+3$	N	N	P	P	P	P



2. $f(x) = \frac{(2-x)(x+4)}{x^2-9}$

x-intercepts(s): ($y=0$)

$0=(2-x)(x+4)$ $(2,0)$ & $(-4,0)$

$2-x=0$; $x+4=0$

$+x$ $+x$

$x=2, -4$

y-intercept: $x=0$

$f(0)=\frac{(2)(4)}{-9} = -\frac{8}{9}$ $(0, -\frac{8}{9})$

Vertical asymptote(s):

$0=(x-3)(x+3)$ $x=3$ & $x=-3$

$x-3=0$; $x+3=0$

$x=3$ $x=-3$

Circle: Proper or Improper deg Num. = deg Denom.

End behavior asymptotes*:

as $x \to -\infty$, $f(x) \to$ -1 $y \approx \frac{-x^2}{x^2}$

as $x \to \infty$, $f(x) \to$ -1 $=-1$

Create a sign line, then complete the graph above.

$2-x$	P	P	P	N	N
$x+4$	N	P	P	P	P
$x+3$	N	N	P	P	P
$x-3$	N	N	N	N	P