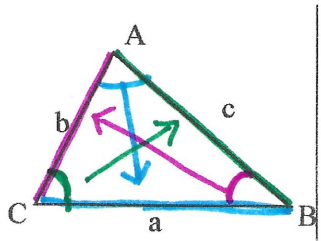
LAW OF SINES AND LAW OF COSINES

Given any ΔABC with sides a, b, and c, then:



LAW OF SINES:

$$\frac{\sin\angle A}{a} = \frac{\sin\angle B}{b} \quad OR$$

$$\frac{\sin\angle A}{a} = \frac{\sin\angle C}{c} \quad OR$$

$$\frac{\sin\angle B}{b} = \frac{\sin\angle C}{c}$$

LAW OF COSINES:

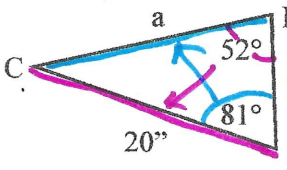
$c^2 = a^2 + b^2 - 2ab(\cos\angle C)$ OR

$b^2 = a^2 + c^2 - 2ac(\cos\angle B)$ OR

$a^2 = b^2 + c^2 - 2bc(\cos\angle A)$

form of Pythagorean Thm — adjustment of areas because there is NO hypotenuse (NO 90° ∠)

- Given 2 angles and a side (AAS or ASA), use Law of Sines:



Law of Sines

$$\frac{\sin 52°}{20} = \frac{\sin 81°}{a}$$

$$\frac{a(\sin 52°)}{(\sin 52°)} = \frac{20(\sin 81°)}{(\sin 52°)}$$

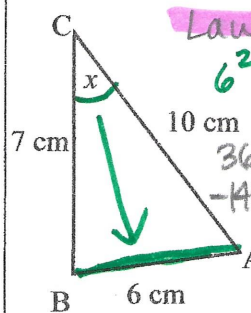
$$a = \frac{20(\sin 81°)}{\sin 52°}$$

Be sure to put EXACT and APPROXIMATE answers! →

$a \approx 25.07$ in

- Given 3 sides (SSS), use Law of Cosines:

Law of Cosines



$6^2 = 7^2 + 10^2 - 2(7)(10)(\cos x)$

$36 = 149 - 140(\cos x)$

$-149 \quad -149$

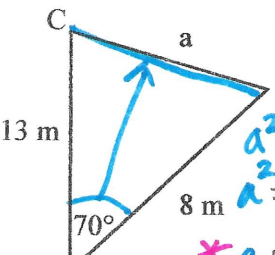
$$\frac{-113}{-140} = \frac{-140(\cos x)}{-140}$$

$\cos x = \frac{113}{140}$

$x = \cos^{-1}\left(\frac{113}{140}\right)$

$x \approx 36.18°$

- Given 2 sides and an included angle (SAS), use Law of Cosines:



Law of Cosines

$a^2 = 13^2 + 8^2 - 2(13)(8)(\cos 70°)$

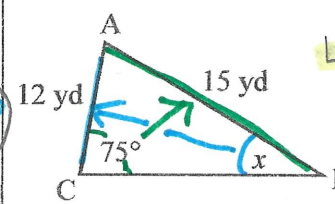
$a^2 = 233 - 208(\cos 70°)$

$a = \sqrt{233 - 208(\cos 70°)}$

$a \approx 12.72$ m

*ONLY need + root because it is a side LENGTH!

- Given 2 sides and a non-included angle (SSA), use Law of Sines:



Law of Sines

$$\frac{\sin 75°}{15} = \frac{\sin x}{12}$$

$$\frac{15(\sin x)}{15} = \frac{12(\sin 75°)}{15}$$

$\sin x = \frac{12(\sin 75°)}{15}$

$x = \sin^{-1}\left(\frac{12(\sin 75°)}{15}\right)$

$x \approx 50.60°$

* Be careful of the ambiguous cases of SSA! Check to see if no triangle is possible, or if one or two triangles are possible.