Graphing and Writing Polynomial Functions

different directions

same directions

End Behavior	Positive Leading Coefficient	Negative Leading Coefficient
Odd Degree Polynomial	as $x \to -\infty$, $f(x) \to$ $-\infty$ as $x \to \infty$, $f(x) \to$ ∞	as $x \to -\infty$, $f(x) \to$ ∞ as $x \to \infty$, $f(x) \to$ $-\infty$
Even Degree Polynomial	as $x \to -\infty$, $f(x) \to$ ∞ as $x \to \infty$, $f(x) \to$ ∞	as $x \to -\infty$, $f(x) \to$ $-\infty$ as $x \to \infty$, $f(x) \to$ $-\infty$

Fundamental Theorem of Algebra: Any polynomial of n degree has n roots.

To sketch the graph a polynomial, determine the end behavior, find all of the roots (including multiplicities and non-real roots), and find the y-intercept.

$P(x) = x(x+2)^2(x^2+9)$

Type: 5th degree polynomial

End behavior: +

as $x \to -\infty$, $f(x) \to -\infty$

as $x \to \infty$, $f(x) \to \infty$

y–intercept: $(0,0)$

$P(0) = 0(0+2)^2(0^2+9)$

$P(0) = 0$

x–intercepts: $(0,0)$ & $(-2,0)$ *Bounce due to mult 2

$0 = x(x+2)^2(x^2+9)$

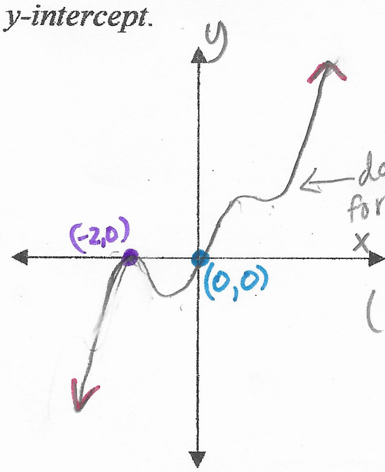
$x = 0 \quad x+2=0 \quad x^2+9=0$

$x = -2 \quad\quad -9 \;\; -9$

$\sqrt{x^2} = \pm\sqrt{-9}$

$x = \pm 3i$

$x = 0, -2, -2, \pm 3i$ ← multiplicity 2



← don't know this for sure... can leave out (preference...)

$f(x) = -3(x-4)^2(x+5)(x+1)$

Type: Quartic (4th degree)

End behavior: −

as $x \to -\infty$, $f(x) \to -\infty$

as $x \to \infty$, $f(x) \to -\infty$

y–intercept: $(0,-240)$

$f(0) = -3(0-4)^2(0+5)(0+1)$

$f(0) = -3(-4)^2(5)(1)$

$= -240$

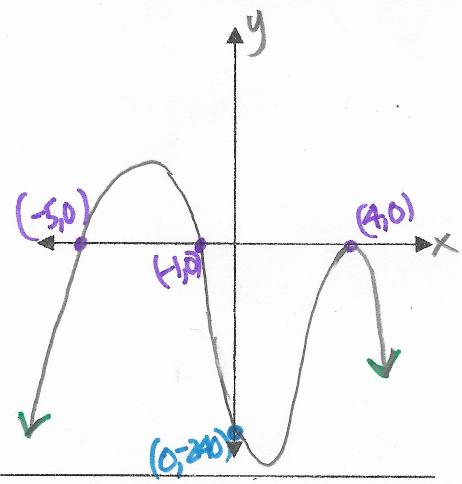
x–intercepts: $(4,0)$*, $(-5,0)$, $(-1,0)$

$0 = -3(x-4)^2(x+5)(x+1)$

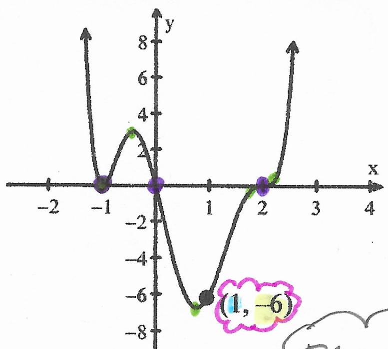
$x-4=0 \quad x+5=0 \quad x+1=0$

$x = 4, 4, -5, -1$

mult. 2



Determine the function that fits the graph.



even degree because ends go same direction. POSITIVE lead coefficient

At least 6th degree because 5 changes in direction

xint: $(-1,0)$, $(0,0)$ & $(2,0)$ through $(1,-6)$

mult 2 (bounce) — mult 3 (curve in, curve out)

Factors: $(x+1)$, x, $(x-2)$

$P(x) = a\,x(x+1)^2(x-2)^3$

$-6 = a(1)(1+1)^2(1-2)^3$

$-6 = a(1)(2)^2(-1)^3$

$-6 = a(1)(4)(-1)$

$\frac{-6}{-4} = \frac{a(-4)}{-4}$

$a = \frac{3}{2}$

$P(x) = \frac{3}{2}x(x+1)^2(x-2)^3$