Logarithms

A logarithm is an inverse to an exponential. This means that the logarithm is the exponent to which another fixed number, the base, must be raised to produce the argument.

$b^x = a$ means the same as $\log_b a = x$

In both versions of this equation, there are some restrictions on the components.

Base: positive values, $b \neq 1$ (cannot be 0 or Negative, or 1)

Exponent: any Real # (no restrictions)

Argument: positive values ($a > 0$; cannot be 0 or Negative)

Examples:

1. $\log_3 27$

$\log_3 3^3$

3

because $3^3 = 27$

2. $\log_4 \frac{1}{256}$

$\log_4 (4^{-4})$

-4

because $4^{-4} = \frac{1}{256}$

3. $\log_{27} \frac{1}{9}$

$\log_{3^3} (3^{-2})$

$-\frac{2}{3}$

$27^x = \frac{1}{9}$

$(3^3)^x = 3^{-2}$

$\frac{3x}{3} = \frac{-2}{3}$

$x = \frac{-2}{3}$

Log Rules:

Log of a **Product Rule:**

$\log_b(zw) = \log_b z + \log_b w$

log of product = sum of logs of factors

$\log_2(8x) = \log_2 8 + \log_2 x$

$= 3 + \log_2 x$

Log of a **Quotient Rule:**

$\log_b\left(\frac{z}{w}\right) = \log_b z - \log_b w$

log of quotient = difference of logs of numerator & denominator

$\log_5\left(\frac{x}{25}\right) = \log_5 x - \log_5 25$

$= \log_5 x - 2$

Log of a **Power Rule:**

$\log_b(z^w) = w \cdot \log_b z$

log of power = product of "exponents"

$\log_7(x^5) = 5 \cdot \log_7 x$