

### Arithmetic and Geometric Means

Finding the MISSING terms in a sequence

#### Arithmetic Means Example

→ repeated addition of some difference

7-4=3 "jumps"

x	3	4	5	6	7	8
f(x)	-2	12	26	40	54	68

-14 +d +d +d +d

To go **BACKWARD** to **PREVIOUS** term, do the **INVERSE** of the forward operation.

In this example → -14

Determine the missing values for this arithmetic sequence. Use an equation to help solve the problem.

x	5	6	7	8	9	10
f(x)	29	20	11	2	-7	-16

+9 +d +d +d +d

If I start with 12 and add a constant difference 3 times, I get 54.

Write an equation for this. Solve it to find the difference.

$$12 + 3d = 54$$

$$-12 \quad -12$$

$$\frac{3d}{3} = \frac{42}{3}$$

$$d = 14$$

Fill in the missing values.

# steps = difference of INPUTS

$$20 + (10-6)d = -16$$

$$20 + 4d = -16$$

$$-20 \quad -20$$

$$4d = -36$$

$$\frac{4d}{4} = \frac{-36}{4}$$

$$d = -9$$

#### Geometric Means Example

repeated multiplication by constant ratio

x	3	4	5	6	7	8
f(x)	2	8	32	128	512	2048

• 4 • 4 • 4 • 4

To get **PREVIOUS TERM** use the **INVERSE** of forward operation

In this ex: • 1/4 or ÷ 4

Determine the missing values for this geometric sequence. Use an equation to help solve the problem.

x	1	2	3	4	5	6
f(x)	4	12	36	108	324	972

• 3 • 3 • 3 • 3

If I start with 8 and multiply a constant factor 3 times, I get 512.

Write an equation for this. Solve it to find the factor.

$$64 \cdot r \cdot r \cdot r = 512$$

$$8 \cdot 8 \cdot 8 = 512$$

$$\frac{8r^3}{8} = \frac{512}{8}$$

$$(r^3)^{1/3} = (64)^{1/3}$$

$$r = 4$$

"Undoing" a power of 3... we want only 1 out of 3 factors in r<sup>3</sup>.

Fill in the missing values.

$$4 \cdot r^{(6-1)} = 972$$

$$\frac{4r^5}{4} = \frac{972}{4}$$

$$(r^5)^{1/5} = (243)^{1/5}$$

$$r = 3$$

\*\*\*When the exponent is EVEN, there are TWO possible solutions (+/-)!