Exponential & Logarithmic Equations

1. $27^{2x} = \left(\frac{1}{243}\right)^{x+3}$

$(3^3)^{2x} = (3^{-5})^{x+3}$

If equivalent powers w/SAME BASE → exponents are =

one to one property of =

$3(2x) = -5(x+3)$

$6x = -5x - 15$

$+5x \quad +5x$

$\frac{11x}{11} = \frac{-15}{11}$

$x = -\frac{15}{11}$

2. $\log_{3x} 54 = 2$

SINGLE LOG → Rewrite in exponential form

$(3x)^2 = 54$

$\frac{9x^2}{9} = \frac{54}{9}$

$\sqrt{x^2} = \pm\sqrt{6}$

$x = \pm\sqrt{6}$

$x = \sqrt{6}$

Check:

$x = \sqrt{6}$ $\log_{+} 54$ ✓

$x = -\sqrt{6}$ $\log_{-} 54$ ✗

* $-\sqrt{6}$ is EXTRANEOUS because the base must be POSITIVE!

3. $\frac{\log(2x-7)}{\log(x+8)} = 1$

$\log(2x-7) = \log(x+8)$

one to one property of =

$2x - 7 = x + 8$

$-x \quad -x$

$x - 7 = 8$

$+7 \quad +7$

$x = 15$

Check:

$\frac{\log 23}{\log 23} = 1$ ✓

* Be sure to check to see that answers are NOT extraneous!!!

4. $\log_7(8 + 3x) = \log_7 x$

If $\log_b x = \log_b y$ → $x = y$

one to one property of =

$8 + 3x = x$

$-3x \quad -3x$

$\frac{8}{-2} = \frac{-2x}{-2}$

$x = -4$

No Solution!

Check:

$\log_7(-4) = \log_7(-4)$

* -4 is extraneous because the argument cannot be NEGATIVE!

5. $2\log_5 x + \log_5 3 = \log_5 48$

Need $\log_b x = \log_b y$

Log of a Power

Log of a Product

$\log_5 x^2 + \log_5 3 = \log_5 48$

$\log_5(3x^2) = \log_5 48$

one to one property of =

$\frac{3x^2}{3} = \frac{48}{3}$

$\sqrt{x^2} = \pm\sqrt{16}$

$x = \pm 4$

$x = 4$

Check

$x = 4$

$2\log_5 4 + \ldots$ ✓

$x = -4$

$2\log_5(-4) + \ldots$

* -4 is extraneous!

6. $4^{x+2} = 250$

Since 4 & 250 can't be rewritten as powers w/ same base, Log BOTH sides (Inverse of exponential) to bring variable out of exponent.

$\log 4^{(x+2)} = \log 250$

Log of Power

$(x+2)\log 4 = \log 250$

$\frac{(x+2)\log 4}{\log 4} = \frac{\log 250}{\log 4}$

$x + 2 = \frac{\log 250}{\log 4}$

$x = \frac{\log 250}{\log 4} - 2$

$x \approx 1.9829$