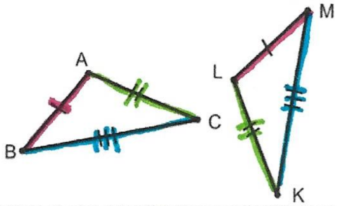
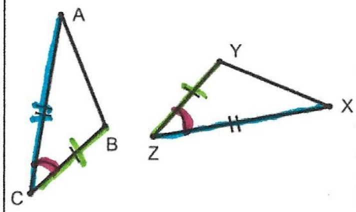
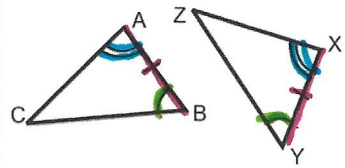
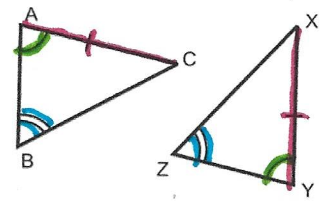
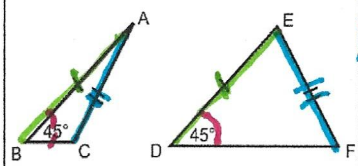
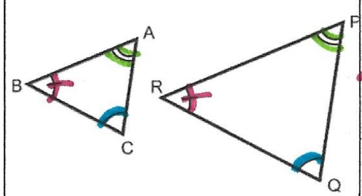
Triangle Congruence Properties

Need at least THREE corresponding pairs of congruent parts to guarantee triangles are congruent!

Guarantee ≅ Triangles!!!

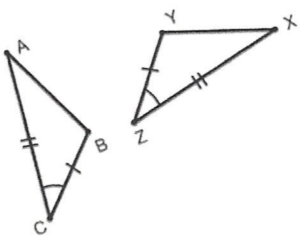
Side-Angle Relationship	Picture	Guarantees Congruence?
SSS *SIDE–SIDE–SIDE* *Three pairs of congruent sides*		$\overline{AB} \cong \overline{LM}$ $\overline{AC} \cong \overline{LK}$ GIVEN $\overline{BC} \cong \overline{MK}$ $\triangle ABC \cong \triangle LMK$ by SSS
SAS *SIDE–ANGLE–SIDE* *Two pairs of congruent sides and one pair of congruent angles (and the angles are between the pairs of sides)*		$\overline{AC} \cong \overline{XZ}$ $\overline{BC} \cong \overline{YZ}$ GIVEN $\angle C \cong \angle Z$ $\triangle CAB \cong \triangle ZXY$ by SAS
ASA *ANGLE–SIDE–ANGLE* *Two pairs of congruent angles and one pair of congruent sides (and the sides are between the pairs of angles)*		$\angle B \cong \angle Y$ $\angle A \cong \angle X$ GIVEN $\overline{AB} \cong \overline{XY}$ $\triangle BAC \cong \triangle YXZ$ by ASA
AAS (or SAA) *ANGLE– ANGLE–SIDE* *Two pairs of congruent angles and one pair of congruent sides (but the sides are NOT between the pairs of angles)*		$\angle A \cong \angle Y$ $\angle B \cong \angle Z$ $\overline{AC} \cong \overline{YX}$ $\triangle ABC \cong \triangle YZX$ by AAS
SSA (or ASS) *Two pairs of congruent sides and one pair of congruent angles (but the angles are NOT between the pairs of sides)*		$\overline{AB} \cong \overline{ED}$ $\overline{AC} \cong \overline{EF}$ $\angle B \cong \angle D$ NO (Not guaranteed congruent) **More than one triangle is possible.** *Need more info!
AAA *Three pairs of congruent angles*		$\angle C \cong \angle Q$ $\angle A \cong \angle P$ $\angle B \cong \angle R$ NO **There is no guarantee the corresponding sides are congruent.** Guaranteed SIMILAR but not necessarily congruent!

DO NOT guarantee ≅ Δ's!!!

More on Triangle Congruence

Once we find three corresponding pairs of congruent parts to prove two triangles are CONGRUENT, then we know ALL SIX corresponding parts of those congruent triangles are congruent!

Using SAS example from "Triangle Congruence Properties" Entry:



Before, we determined that:

$\overline{AC} \cong \overline{XZ}$

$\angle C \cong \angle Z$

$\overline{BC} \cong \overline{YZ}$

so, $\Delta ABC \cong \Delta XYZ$ by SAS.

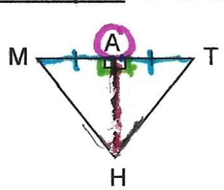
Since the triangles are guaranteed to be congruent, then we can now guarantee that $\angle A \cong \angle X$, $\angle B \cong \angle Y$, and $\overline{AB} \cong \overline{XY}$ because congruent triangles have congruent corresponding parts.

($\cong\Delta s \rightarrow \cong$ parts)

A few additional things may come up when trying to prove triangles congruent:

- Sometimes two triangles can SHARE a side. Even if they are not marked, these pieces can be guaranteed congruent because anything is congruent to ITSELF. In math, we call this the Reflexive Property.
- Sometimes in order to be clear about which specific angle we are referring to, we need to use a 3 letter name instead of a single letter. This is necessary when you have more than one angle with the same vertex.

Example: Is $\Delta MAH \cong \Delta TAH$?



There are two pieces marked:

$\overline{MA} \cong \overline{TA}$

$\angle MAH \cong \angle TAH$ (To say $\angle A \cong \angle A$ would not be correct because they are NOT the exact same piece!) Vertex MUST be in Middle!

Only two congruent corresponding parts is NOT enough information to guarantee $\cong$ triangles!

But, we also know $\overline{AH} \cong \overline{AH}$ by Reflexive Property

So we can conclude $\Delta MAH \cong \Delta TAH$ by SAS.