

## VOLUME AND TOTAL SURFACE AREA OF PRISMS, CYLINDERS, PYRAMIDS, AND CONES

Use the 4 steps for “area sub-problems”:

1. Picture Equation
2. Formulas
3. Simplify
4. Answer (exact and approximate)

### PRISMS:

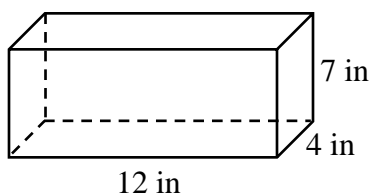
$$\text{Volume} = (\text{Area of Base}) \bullet \text{Height}$$

Total surface area is the sum of the areas of all faces (or sides)

*NOTE: The base is NOT always located on the “bottom” of prisms!!!*

The bases of a prism are the two congruent faces that are on opposite sides of the prism, and these two faces are connected by parallelograms, rectangles, or rhombi.

#### Volume & TSA of a Rectangular Prism



$$V = (\text{Area of Base}) \bullet \text{Height}$$

$$V = \left( 7 \begin{array}{|c|} \hline \square \\ \hline 12 \end{array} \right) \bullet 4$$

$$V = (12 \bullet 7) \bullet 4$$

$$V = 336 \text{ in}^3$$

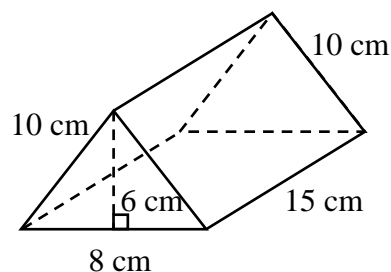
$$\text{TSA} = 2 \left( 7 \begin{array}{|c|} \hline \square \\ \hline 12 \end{array} \right) + 2 \left( 4 \begin{array}{|c|} \hline \square \\ \hline 12 \end{array} \right) + 2 \left( 4 \begin{array}{|c|} \hline \square \\ \hline 7 \end{array} \right)$$

$$\text{TSA} = 2(12 \bullet 7) + 2(12 \bullet 4) + 2(7 \bullet 4)$$

$$\text{TSA} = 168 + 96 + 56$$

$$\text{TSA} = 320 \text{ in}^2$$

#### Volume & TSA of a Triangular Prism



$$V = (\text{Area of Base}) \bullet \text{Height}$$

$$V = \left( \begin{array}{|c|} \hline \triangle \\ \hline 8 \quad 6 \\ \hline 8 \end{array} \right) \bullet 15$$

$$V = \left( \frac{1}{2} \bullet 8 \bullet 6 \right) \bullet 15$$

$$V = (24) \bullet 15$$

$$V = 360 \text{ cm}^3$$

$$\text{TSA} = 2 \left( \begin{array}{|c|} \hline \triangle \\ \hline 8 \quad 6 \\ \hline 8 \end{array} \right) + 2 \left( 10 \begin{array}{|c|} \hline \square \\ \hline 15 \end{array} \right) + 8 \begin{array}{|c|} \hline \square \\ \hline 15 \end{array}$$

$$\text{TSA} = 2\left(\frac{1}{2} \bullet 8 \bullet 6\right) + 2(15 \bullet 10) + 2(15 \bullet 8)$$

$$\text{TSA} = 48 + 300 + 240$$

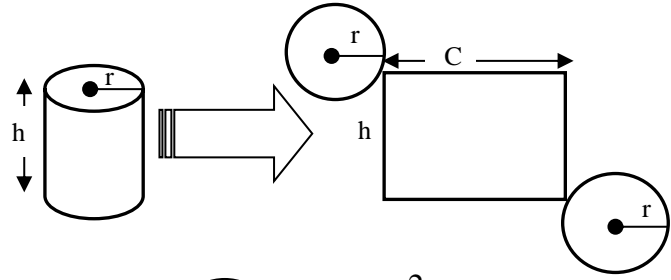
$$\text{TSA} = 588 \text{ cm}^2$$

**CYLINDERS:**

$V = (\text{Area of Circular Base}) \bullet \text{Height}$

The surface area of a cylinder is the sum of the areas of the top and the bottom base (two circles) and the curved side (a rectangle).

- The bases of a cylinder are equal in area.
- The height of the cylinder is the height of the rectangle.
- The circumference of the cylinder is the length of the rectangle.



$TSA = 2 \text{ (circle with radius } r) + h \text{ (rectangle with length } 2\pi r)$

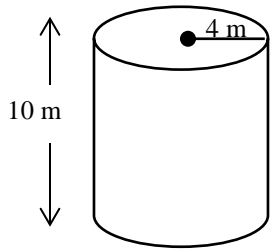
$TSA = 2 \text{ (circle with radius } 4) + 10 \text{ (rectangle with length } 2\pi \bullet 4)$

$TSA = 2(\pi \bullet 4^2) + 10(2\pi \bullet 4)$

$TSA = 32\pi + 80\pi$

$TSA = 112\pi \text{ m}^2$

$TSA \approx 351.86 \text{ m}^2$



$V = (\text{Area of base}) \bullet \text{Height}$

$V = \text{(circle with radius } 4) \bullet 10$

$V = (\pi \bullet 4^2) \bullet 10$

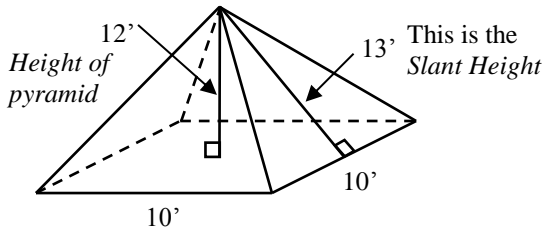
$V = 160\pi \text{ m}^3$

$V \approx 502.65 \text{ m}^3$

**PYRAMIDS:**

$V = \frac{1}{3} (\text{Area of Base}) \bullet \text{Height}$

TSA = sum of the areas of all faces



$V = \frac{1}{3} \left( \begin{matrix} \square \\ 10 \end{matrix} \right) \bullet 12$

$V = \frac{1}{3} (10 \bullet 10) \bullet 12$

$V = 400 \text{ ft}^3$

$TSA = 4 \left( \begin{matrix} \triangle \\ 10 \end{matrix} \right) + 10 \begin{matrix} \square \\ 10 \end{matrix}$

$TSA = 4 \left( \frac{1}{2} \bullet 10 \bullet 13 \right) + (10 \bullet 10)$

$TSA = 260 + 100$

$TSA = 360 \text{ ft}^2$

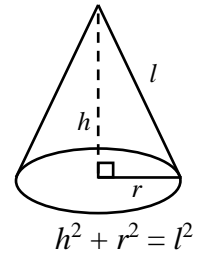
**CONES:**

$V = \frac{1}{3} (\text{Area of Circular Base}) \bullet \text{Height}$

TSA = (Area of Base) + (Lateral Surface Area)

$TSA = \text{(circle with radius } r) + \text{(sector with radius } l)$

$TSA = \pi r^2 + \pi r l$

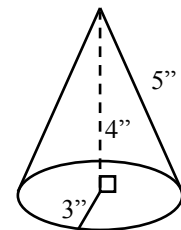


$V = \frac{1}{3} \text{(circle with radius } 3) \bullet 4$

$V = \frac{1}{3} (\pi \bullet 3^2) \bullet 4$

$V = 12\pi \text{ in}^3$

$V \approx 37.70 \text{ in}^3$



$TSA = \text{(circle with radius } 3) + \text{(sector with radius } 5)$

$TSA = (\pi \bullet 3^2) + (\pi \bullet 3 \bullet 5)$

$TSA = 9\pi + 15\pi$

$TSA = 24\pi \text{ in}^2$

$TSA \approx 75.40 \text{ in}^2$